# Self-focusing distance of very high power laser pulses

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**Abstract:** We show numerically for continuous-wave beams and experimentally for femtosecond pulses propagating in air, that the collapse distance of intense laser beams in a bulk Kerr medium scales as $1/P^{1/2}$ for input powers P that are moderately above the critical power for self focusing, but that at higher powers the collapse distance scales as 1/P.



**OCIS codes:** (190.5530) Pulse propagation and solitons; (190.7110) Ultrafast nonlinear optics; (010.1300) Atmospheric propagation

## References and Links

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## 1. Introduction

In 1965, Kelley [1] predicted that an intense laser beam that propagates in a bulk Kerr medium would undergo catastrophic collapse, provided its input power P exceeds the critical power for collapse $P_{cr}$ and that the distance to the point of collapse $L_{col}$ would scale as $1/P^{1/2}$. These predictions were confirmed in numerous experiments since (see, e.g., Ref. [2] and the references therein). A few years later it was predicted theoretically, and observed experimentally for continuous-wave (cw) beams propagating in $CS_2$, that at sufficiently high input powers the collapse distance scales as 1/P [3,4]. More recently, the self-focusing dynamics of high-power ultrashort laser pulses has attracted significant attention, particularly


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for the case in which high-power ultrashort pulses propagate in air [5-8]. The resulting long-distance filamentation behavior has been exploited for remote sensing in the atmosphere [9]. Despite the extensive numerical simulations that have been undertaken, few analytic treatments have been performed that provide a deep understanding of the observed dynamics. In this Letter we present a new derivation of the 1/P law, which is more intuitive and less technical than the one in Ref. [4]. This derivation allows us to estimate the value of a second power threshold $P_{MF}$, such that the $1/P^{1/2}$ scaling holds for $P \ll P_{MF}$ and the 1/P scaling holds for $P \gg P_{MF}$. We also present the first numerical simulations that show the transition from the $1/P^{1/2}$ regime to the 1/P regime. Although our derivation and simulations, as well as the experimental results of Ref. [3], are for cw beams, we provide experimental evidence that these results are also valid for ultrashort pulses, by observing both the $1/P^{1/2}$ and the 1/P regimes for femtosecond pulses propagating in air.

## 2. Theory

The difference in the dynamics between collapsing ``low-power'' ($P_{cr} \ll P \ll P_{MF}$) and ``high-power'' ($P \gg P_{MF}$) beams is due to the effect of input beam noise. This effect was first analyzed by Bespalov and Talanov (BT) [10], who showed that *plane-wave* solutions of the nonlinear Schrödinger equation (NLS)

$$iA_z(x,y,z) + \Delta A + |A|^2 A = 0, \quad A(0,x,y) = A_0(x,y) \tag{1}$$

are modulationally unstable (here A is the beam amplitude, z is the axial distance, and x and y are the transverse coordinates). Since then, the BT instability analysis was interpreted as showing that noise leads to breakup of powerful collapsing beams into multiple filaments (MF). Recently, however, Fibich and Ilan showed that the solution of the NLS (1) with a Gaussian input beam with $P=15P_{cr}$ and 10% noise does not become modulationally unstable, but rather collapses to a single filament at nearly the same location as in the absence of noise [11]. Indeed, subsequent experiments [12], as well as rigorous analysis [13] have shown that noisy input beams, whose power is moderately (but not highly) above $P_{cr}$, collapse with the cylindrically-symmetric, self-similar Townes profile. Of course, such a noisy beam would subsequently breakup into MF if a collapse-arresting mechanism, such as nonlinear saturation or plasma defocusing, is added to the NLS model [4]. In that case, the beam would initially collapse (at $L_{col}$) with a single filament that would undergo several focusing-defocusing cycles before breaking into MF. Therefore, the initial collapse distance $L_{col}$ would be unaffected by noise, i.e., $L_{col} \approx L_{SF} \sim 1/P^{1/2}$, where $L_{SF}$ is the self-focusing distance in the absence of noise.

In order to understand the "failure" of the BT analysis for noisy Gaussian beams with $P=O(15P_{cr})$, we note that the key property of plane waves which is used in the BT analysis is that they undergo self-phase modulation (SPM) without any self-focusing. This allows the unstable modes to have a sufficient propagation distance over which they can grow without being "interrupted" by the large-scale transverse self-focusing dynamics. In order for this to occur for a Gaussian beam, the condition $L_{SPM} \ll L_{SF}$ must be met, where $L_{SPM}$ is the characteristic distance for SPM. Since LSPM~1/P and LSF~$1/P^{1/2}$, the condition $L_{SPM} \ll L_{SF}$ would always hold above a certain power threshold $P_{MF}$. Thus, if $P \ll P_{MF}$ the beam initially collapses to a single filament at $L_{col} \approx L_{SF} \sim 1/P^{1/2}$, and it may (or may not) break later into MF. If, however, $P \gg P_{MF}$, the unstable modes can grow over a distance of $L_{SPM}$, hence the beam would break into collapsing MF. Therefore, in this case $L_{col} \approx L_{SPM} \sim 1/P$, as was first predicted in [3,4].

## 3. Simulations

In our simulations we first solve the NLS (1) with clean input Gaussians beams $A_0(x,y) = 2P^{1/2}\exp(-x^2-y^2)$. We define the collapse distance $L_{col}$ as the location where the maximal amplitude $\max_{x,y}|A|$ has increased by a factor of 2 from its initial value, and let $S_{col} = S(z= L_{col})$ denote the accumulated phase at the onset of the collapse. Until the beam begins to collapse

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(i.e., for $0 \le z \le L_{col}$), the on-axis phase can be well-approximated by $S(z) \approx z|A_0(0,0)|^2$ (see Fig. 1). Thus, for example $S_{col} \approx 3$ and 16 when $P=10P_{cr}$ and $P=300P_{cr}$, respectively. When 10% random noise is added to the beam, there is little effect for $P=10P_{cr}$, but for $P=300P_{cr}$ the collapse occurs at a much smaller value of z, when the accumulated phase is only $S_{col} \approx 3$ (Fig. 2). Thus, the same noise realization has almost no effect at "low" powers but has a large effect at "high" powers.

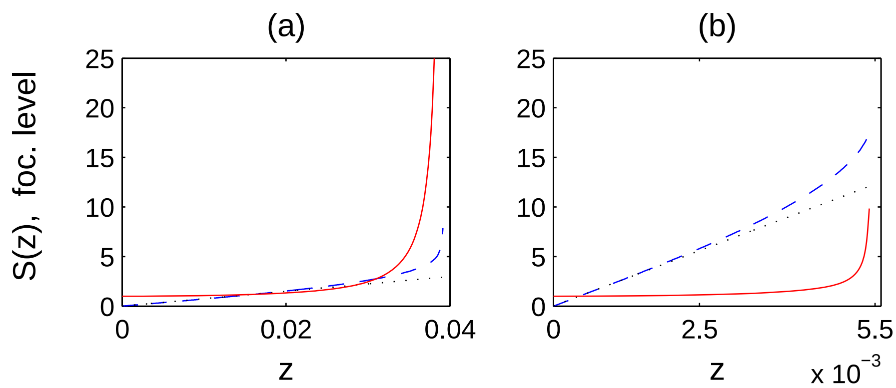


Fig. 1. On-axis phase S(z) (dashes) and focusing level $\max_{x,y}|A(z,x,y)|/|A_0(0,0)|$ (solid) of the solution of the NLS (1) with noiseless Gaussian input beam. Dotted line is $z|A_0(0,0)|^2$. (a) $P = 10P_{cr}$ (b) $P = 300P_{cr}$

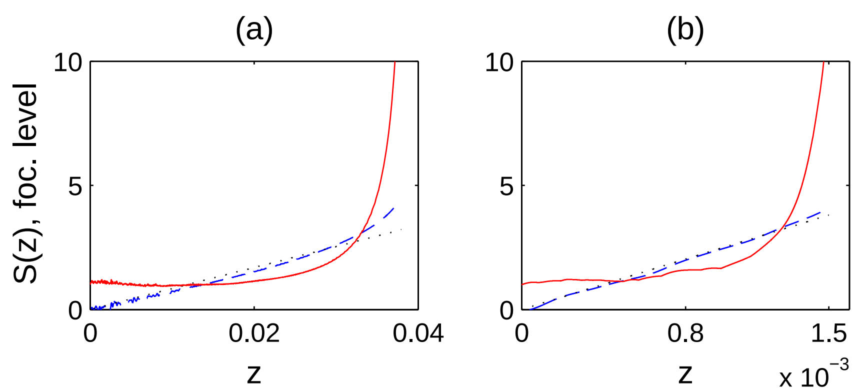


Fig. 2. Same as Fig. 1 with 10% random noise.

We can make a quantitative estimate for the power threshold $P_{MF}$ as follows. For the case of clean beams, $S_{col}= S(z= L_{col}) \sim P^{1/2}$ since $L_{col} = L_{SF} \sim 1/P^{1/2}$ and since $S(z) \approx z|A_0(0,0)|^2$. Indeed, numerical fit of the values of $S_{col}$ for $10P_{cr} \le P \le 600P_{cr}$ yields $S \sim 0.34(P/P_{cr})^{0.47}$, as shown in Fig 3. In the presence of 10% noise, $S_{col}$ displays similar behavior for $P \le 40P_{cr}$, since noise has a negligible effect in this power regime. However, for $P>40P_{cr}$, noise accelerates the collapse. As a result, $S_{col}$ is slowly monotonically decreasing for $P>40P_{cr}$. We thus see that the collapse occurs for $3<S_{col}<5$. Since $S(z) \approx z|A_0(0,0)|^2$, we define $L_{SPM}=S_{col}/|A_0(0,0)|^2 \approx 4/|A_0(0,0)|^2=1/P$. In addition, since for (clean) Gaussian beams $L_{SF} \sim 0.11/P^{-0.53}$, see Fig. 4, the condition $L_{SPM} < L_{SF}$ is satisfied for $P> P_{MF} \sim 100P_{cr}$.

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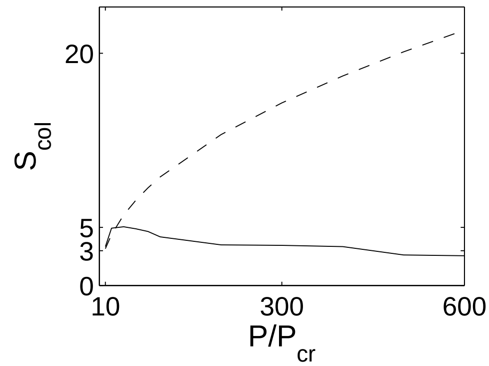


Fig. 3. On-axis phase at the onset on collapse, as a function of $P/P_{cr}$, for clean beams (dashed line) and for beams with 10% noise (sold line).

In order to see the transition in the collapse distance from $1/P^{1/2}$ to $1/P$, we solved the NLS (1) with Gaussian initial conditions, calculated the collapse distance $L_{col}$ as a function of input power at various noise levels, and found the best-fitting power law $L_{col}{\sim}c(P/P_{cr})^b$ (Fig. 4). In the absence of noise, $L_{col}{\sim}\ 0.11P^{-0.53}$ for $4 \le P/P_{cr} \le 600$, in agreement with the b=-1/2 theoretical prediction. When we add a 10% random noise, the collapse distance remains nearly unchanged for $P \le 40P_{cr}$. At higher powers, noise leads to a considerable reduction in $L_{col}$, which scales as $P^{-1.18}$, in reasonable agreement with the b=-1 analytic prediction. We can get a better agreement with the 1/P prediction by lowering the noise level. For example, the addition of 5% random noise, using the same noise realization as before, gives b=-1.1 (data not shown). In Fig. 5 we show the spatial profile of the beam just as it begins to collapse. Although we used the same noise realization in all cases, a single filament is formed for $P = 20P_{cr}$, several filaments are observed for $P = 40P_{cr}$, and numerous filaments are formed for $P = 150P_{cr}$. This confirms that the transition to the 1/P regime occurs when MF are formed during the initial stage of collapse.

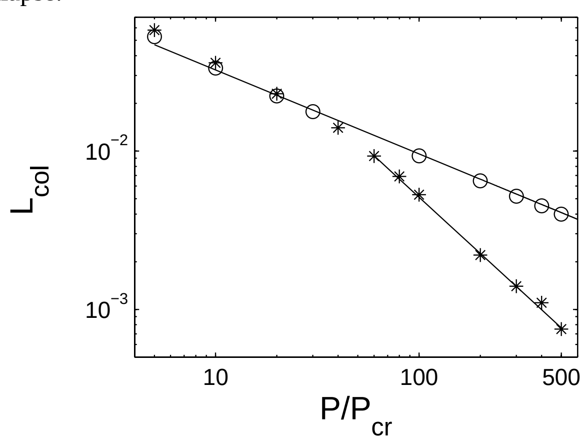


Fig. 4. Collapse distance $L_{col}$ as a function of input beam power (simulations). (o) – no noise, (*) - 10% random noise. Solid lines are the best fitting power laws.

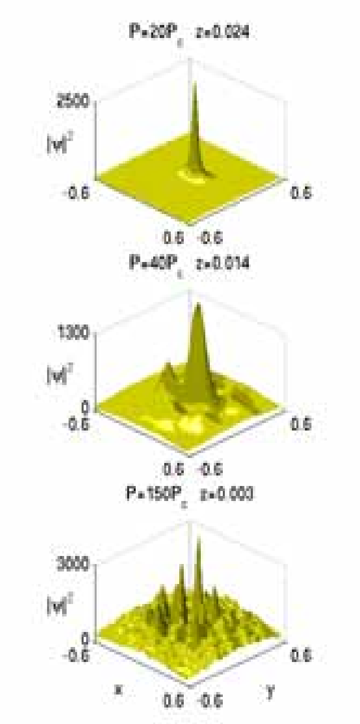


Fig. 5. Spatial profile Gaussian beams with 10% random noise at various input powers as they begin to collapse (simulations).

## 4. Experiments

In our experiments, we determined the collapse distance $L_{col}$ as a function of input beam power. The experiments were conducted using 45-fsec laser pulses at 800nm, and the beam diameter was 35 mm. The peak power of the pulse was varied without changing the spatial profile by using a $\lambda/2$ plate and a polarizer. The lower power experiments were conducted using a single amplification stage, resulting in a relatively low level of spatial noise, whereas for the higher power experiments we also used a second four-pass amplifier which added distortion to the laser beam. The laser pulse propagated in air ($P_{cr}$ ~3-5 GW) within the lab. The collapse distance at each power level was defined as the shortest propagation distance at which the laser beam could create a visible damage to a Polyvinyl Chloride (PVC) target, whose damage threshold for 50 fsec pulses is about $10^{13}$ W/cm$^2$.

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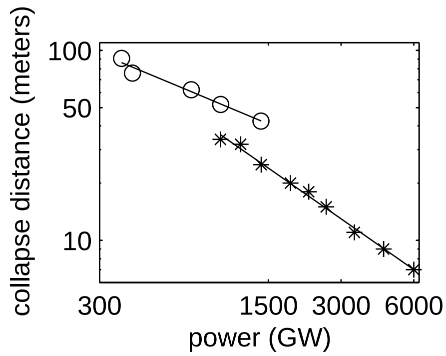


Fig. 6. The collapse distance $L_{col}$ as a function of input power P (experimental). Circles and stars represent data where a single amplifier and two amplifiers were used, respectively. Solid lines are the best-fitting power laws. Note the similarity to Fig. 4.

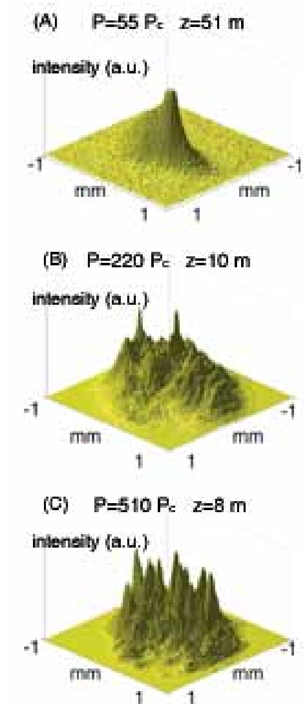


Fig. 7. Spatial beam profile as it begins to collapse (experimental). Note the similarity to Fig. 5.

The experimental results are presented in Fig. 6. In the first series of experiments we used the single amplifier set-up, hence the input profile had a relatively low noise level. The PVC damage patterns indicated a single filament (Fig. 7A), and the power law fit $L_{col}$~$c(P/P_{cr})^b$ yielded b = -0.53. When we added the second amplifier, we increased the power range, and also introduced considerable asymmetric noise to the beam. As a result, in these experiments the PVC damage patterns always displayed MF patterns (Fig. 7B-C); the number of filaments increased with the power, and a power law fit of the collapse distance gave b = -0.89. Note that, unlike in the simulations, in the experiments there is less control over the noise level and pattern which can change from shot to shot, hence it is harder to ``approach'' the b= -1 theoretical prediction.

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The experimental results thus confirm the existence of two distinct self-focusing regimes. For ``low” powers ($P<<P_{MF}$), only a single filament is formed initially, and the collapse distance scales as $L_{col} \approx L_{SF}$~$1/P^{1/2}$. For higher input powers ($P>>P_{MF}$), multiple filaments are formed as soon as the beam begins to collapse, hence $L_{col} \approx L_{SPM}$ ~1/P. The overlap in the power regimes between the two sets of experiments show that 1) noise can lead to a considerable reduction of the collapse distance, and 2) that the threshold power $P_{MF}$ decreased substantially as a result of the high noise introduced by the second amplifier. This could be expected, since a very noisy beam needs much less than $S_{col} \approx 4$ in order for the instabilities to grow to the level where they lead to MF.

## 5. Conclusion

In conclusion, in this study we provide the first numerical demonstration that in the presence of input beam noise, the self-focusing distance scales as $1/P^{1/2}$ for input powers that are moderately above $P_{cr}$ but scales as 1/P at much higher input powers, and that the transition to the 1/P regime occurs when noise leads to MF of the beam before it collapses. Although the theory was developed for cw beams, there was excellent agreement between the simulations for cw beams and the experimental results for femtosecond pulses propagating in air, due to the minimal contribution of air dispersion before the pulse undergoes collapse.

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